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COMMENT

A Dirac-like equation for the photon

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Abstract. It has been shown recently by Jena *et al* that a Dirac-like equation for the photon can be obtained as the zero mass limit of the Duffin–Kemmer–Petian equation for massive spin-one particle with appropriate identification of the DKP spinor with electromagnetic field strengths and potentials. It is the purpose of this comment to show that a Dirac-like equation for the photon can be obtained in a straightforward manner, without invoking either the DKP equation or the complex combination of the electric and magnetic fields.

We first consider Maxwell's equations in free space without sources. It is sufficient to take (in Gaussian units with $c = 1$)

$$\nabla \times E = -\partial H/\partial t \qquad \nabla \times H = \partial E/\partial t.$$

Since $\nabla \cdot E = \nabla \cdot H = 0$ follows from these if they are imposed as conditions at $t = 0$.

If we introduce

$$\psi = \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix} \qquad \beta^1 = \left(\begin{array}{c|c} 0 & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} \\ \hline \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{matrix} & 0 \end{array}\right)$$

$$\beta^2 = \left(\begin{array}{c|c} 0 & \begin{matrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{matrix} \\ \hline \begin{matrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{matrix} & 0 \end{array}\right) \qquad \beta^3 = \left(\begin{array}{c|c} 0 & \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \hline \begin{matrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & 0 \end{array}\right)$$

and

$$\beta^0 = \left(\begin{array}{c|c} \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix} & 0 \\ \hline 0 & \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{matrix} \end{array}\right)$$

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then the two Maxwell's curl equations become

$$\beta^{\mu}\,\partial_{\mu}\psi = 0$$

where $\partial_0 = \partial/\partial x_0 = \partial/\partial t$, $\partial_i = \partial/\partial x_i$, $i = 1, 2, 3$ and the summation convention has been used on $\mu\,(\mu = 0\text{–}3)$.

In free space with sources, the two Maxwell's curl equations are

$$\nabla \times E = -\partial H/\partial t \qquad \nabla \times H = \partial E/\partial t + 4\pi j$$

$\nabla \cdot E = 4\pi\rho$ and $\nabla \cdot H = 0$ follow from these and the conservation equation $(\nabla \cdot j + \partial\rho/\partial t = 0)$ if they are imposed as conditions at $t = 0$.

If we further introduce

$$J = \begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

the two Maxwell's curl equations with sources become

$$\beta^{\mu}\,\partial_{\mu}\psi = 4\pi J.$$

It is easily verified that the above equation and its adjoint follow from a Euler–Lagrange variational principle using the following Lagrangian

$$\mathcal{L} = -\tfrac{1}{4}(\bar{\psi}\beta^{\mu}\,\partial_{\mu}\psi - \partial_{\mu}\bar{\psi}\beta^{\mu}\psi) + 2\pi(\bar{\psi}J + \bar{J}\psi).$$

Reference

Jena P K, Naik P C and Pradhan T 1980 *J. Phys. A: Math. Gen.* **13** 2975